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Targeted policy design in transportation: the case of the ferry market*

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Abstract

We model a ferry market where passengers are heterogeneous in their valuation of waiting time and, unlike in previous studies, can take services from all operators. Analyzing their behavior when two operators are active, each providing one service, we find that complex patterns of product differentiation emerge between two goods that (i) do not exactly correspond to the available services and (ii) display service frequencies as quality attributes. A (low-quality) basic good, coinciding with the cheaper service, attracts low-time-value passengers. A (high-quality) composite good, which is a bundle of the two available services, appeals to high-time-value passengers. Consequently, demand is positive for either operator so that an inefficient operator is not crowded out. In the specific case of a mixed duopoly, a price-aggressive public operator spans discipline over (but does not monopolize) the whole market; a soft one boosts "quality" (i.e., frequency) vis-à-vis fraction of the population only, that is yet larger than under classical vertical differentiation. Policy-makers pursuing redistribution objectives should target the cheaper service, in general, privileging either a raise in its frequency (when it is low) or a cut in its price (when frequency is high), depending upon the group of passengers they wish to support.

Keywords: Ferry market; Price and frequency; Time value; Passenger allotment; Product differentiation; Redistribution

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1 Introduction

It is a truism that travel behaviour is complex and multi-faceted. It is also a platitude that price is far from being the unique determinant of travel demand. Travel is mostly a derived demand and some of its determinants are even unrelated to transport operators. Others, on the contrary, directly follow from policy choices, or from the decisions made by transport operators. When appraising the performance of a (passenger) transportation system, it is of primary importance to look beyond the sole prices and account for these determinants as well.

Since the very first contributions attempting to measure travel demand (see, in particular, McFadden [14]), it has been recognized that two determinants of primary importance are (on-vehicle) travel time and service frequency. Depending upon the transportation modes, on-vehicle travel time is essentially related to the characteristics of the transportation network as well as to technological choices. To some extent, it can be viewed as a given attribute. By contrast, service frequency reflects operator(s)' decisions nearly in the same manner as price does. It is thus natural to question jointly their optimality.

Mohring [15] argues that, if users' waiting time is included in the costs attached to transportation services, then the latter exhibit increasing returns to scale so that frequencies are likely to be lower than socially optimal. In monopolistic transportation markets, service frequency is substantially equivalent to any other quality attribute. Hence, in those environments, frequency and quality can be regulated in similar fashions, as Billette de Villemeur [3] points out. It would be tempting to extend the policy conclusions that ensue from these models to more complex frameworks. For example, in oligopolistic mixed markets, where a public and a private operator are competing, one could recommend the quality-adjusted price-cap proposed in Bergantino *et alii* [2] to enhance travellers' welfare. However, details matter.

As evidenced by Spence [20], whether a monopoly provides a lower or a bigger quality than socially optimal it depends upon how the marginal and the average (or representative) consumer compare. Resting on this, van Reeven [21] distinguishes the case in which travellers do not know the timetable, hence take the first available service, from that in which individuals are actually based on the timetable to plan their trips. In the former case, the number of connections supplied by a profit-maximising monopolist is socially optimal, given the price. In the latter case, it is not. Similarly, the very fact that, in oligopolistic markets, travellers are not necessarily bound to a single operator, but may swing from one to the other, does alter the core nature of competition. Therefore, while, at a first glance, moving from monopoly to oligopoly may appear to add gratuitous complexity to the analysis, there are in fact interesting lessons to be drawn, within that framework, to policy-makers' good use.

There is a growing awareness that the design of sound policies cannot spare reference to the peculiarities of the contexts to which they are targeted. For example, the knowledge of cross-elasticities is essential for the design of policies aimed at affecting modal split. Yet, their values depend upon many parameters, including travel motives and even mode shares. This makes it difficult, *a priori*, to rely upon estimates obtained in frameworks other than the concerned one (Acutt and Dodgson [1]). Paulley *et alii* [18] find that transit fare elasticities are more than four times larger for car owners than for users of the public transit system. Hensher [9] evidences significant differences in fare elasticities across fare regimes. To stick to the issue of modal split, while it is by now pretty clear that a uniform decrease in tariffs would be an inefficient way to promote the use of public transportation in urban areas, this paves the way to the adoption of more efficient policies, targeting specific subsets of travellers. Conversely, for any given policy change, one should question who will benefit and who will suffer from it. As policy interventions are often motivated by redistribution goals, it would be especially useful to be able to classify users according to their income. In practice, it is seldom the case that policies can be conditioned on users' income, be it due to information lack, legitimacy problems or simply costs. Yet, the value of time is arguably related to income, together with other characteristics that are also relevant for policy purposes.[1] This means that policy-makers can refer to the value of waiting time as to a reasonably close indicator of passengers' economic conditions.

Coherently with this view, our work investigates the behaviour of passengers who display different valuations of waiting time in multi-service transportation systems, specifically referring to ferry markets. At this aim, a simple model is constructed to capture two main features. First, two operators are active in the market, each providing a service characterized by a price and a frequency. Second, heterogeneous passengers can either patronize one specific service or use both services. Our contribution is twofold. First, we identify the segmentation and, hence, the kind of product differentiation that arise in those markets. Second, and consequently, we draw policy insights and discuss redistribution implications for the population of heterogeneous travellers.

Implicitly, availability of two services displaying price and frequency attributes is meant to reflect a duopolistic sector where operators provide different services engaging in price and frequency competition. Yet, we model neither the characteristics of the firms nor the specific kind of competition they undertake. This is not essential to the very scope of our work. Indeed, in the context that we consider, key determinants are passenger decisions. Despite the stylistic simplicity, our model does allow us to reply the research questions that we raise.

We find that, because passengers can use transport services of both operators, complex patterns of product differentiation arise, which cannot be identified with classical vertical differentiation.[2] Passengers who have low time value, and thus attach little importance to frequency, always use the cheaper service, whether it is more or less frequent. Those who have high time value, and thus benefit much from frequency, use a bundle of the services that the two operators supply. As compared to the cheaper service, this bundle can be viewed as a more expensive and more frequent service. This involves that, in a duopolistic transport market, it is only at

[1] For a review on the value of travel time see, for instance, Wardman [23].

[2] The study of oligopolies in which firms offer products of different qualities traces back to Gabszewicz *et al.* [8]. An exhaustive characterization of product vertical differentiation in duopolistic markets can be found in Wauthy [24].

the aggregate level that frequency can be assimilated to a standard quality dimension. Yet, even this similarity is loose because the bundle accrues to a larger portion of the population than a high-price high-quality product would in a vertically differentiated industry.

To investigate how passengers allot between services, we follow the approach developed in the literature about traffic-flow predictions (see, for instance, Leurent [12]). According to that approach, each individual chooses the travel option that yields the lowest generalized cost (or price) per trip. This cost is directly related to both the monetary price paid for the trip and the value personally attached to the time spent in the wait for the connection. By contrast, it is inversely related to the frequency of service provision.

As far as the adoption of this approach is concerned, our model is akin to that of Yang *et alii* [22]. They investigate how different distributions of time value in the passenger population affect price-and-frequency competition between vertically differentiated bus services, taking into account that individuals can alternatively use the car. To study this issue, two important assumptions are introduced. First, each service displays an exogenously given quality attribute, namely travel time. Second, individuals travel only one journey, hence they can use only one of the three available options. These turn out to be slow bus services supplied at a low price (a low-price-and-quality good), fast bus services supplied at a higher price (a medium-price-and-quality good), and expensive car services (a high-price-and-quality good). Under the two assumptions, the market is segmented as it is usual in a context of vertical differentiation. That is, individuals with low, medium and high time value patronize the low-, medium- and high-price-and-quality good, respectively. The frequency dimension that is also present for bus services does not affect the nature of product differentiation that arises in the market.

Similar market segmentation obtains in Cantos-Sànchez *et al.* [4]. In the mixed duopoly that they consider, two transportation modes, namely bus and train, are available. Like Yang *et alii* [22], they take each individual to travel only one journey, hence to use only one of the available services, and each service to display an exogenously given quality attribute. This determines a unanimous service ranking in the population of passengers, who all bear the same disutility from travel delay. Hence, it brings about vertical differentiation and leads to the standard demand segmentation between low- and high-quality mode. On top of that, each service is assumed to be characterized by a frequency attribute. This is meant to represent supply of a multiplicity of products in the market. Thus, the role of the frequency dimension is that of introducing horizontal differentiation, in its classical notion, into the model.

Our work diverges from that of Yang *et alii* [22] and Cantos-Sànchez *et al.* [4] in that unequally impatient individuals are allowed to travel more than one journey and, more importantly, to use both of the services supplied. This suffices to ensure that low-time-value individuals accrue to the cheaper service (a low-price-and-quality good) and high-time-value individuals to a bundle of the available services (a composite high-price-and-quality good). Thus, as compared to Yang *et alii* [22] and Cantos-Sànchez *et al.* [4], our approach evidences a more sophisticated passenger behaviour and a more nuanced market segmentation, even when additional service attributes such as travel speed are put aside. It is precisely to emphasize this aspect that, in our model, we take quality dimensions not to affect the unanimous ranking of the services provided by the two operators, hence to be neutral with respect to the resulting market segmentation.

While Billette de Villemeur [3] refers to passenger air transport, Yang *et alii* [22] to bus services and Cantos-Sànchez *et al.* [4] to bus and train services, we concentrate on passenger ferry transport. Actually, with some qualifications, our analysis would easily apply to any available pair of public transport modes. Yet, the ferry industry is a particularly appropriate illustration for two main reasons. Firstly, to emphasize the object of our analysis, we represent situations in which no alternative mode is available and the market is entirely covered. This is the case in several Italian, Greek and Scottish islands, which are too small to host airports and sufficiently distant from the mainland that no bridge and channel for railways and/or highways can be built and operated economically and safely. In those islands, all potential passengers are captive to the ferry mode. Secondly, in several real-world contexts, ferry sectors have been experiencing a process of liberalization in recent decades. Within the European Union, entry by new providers has been registered in former (generally public) monopolies, following to the enforcement of the EU Regulation 3577/92 [7] that extended service freedom to coastal navigation and short-hauls connections. Furthermore, in Hong Kong, competition has recently developed between low-capacity ferries, which provide expensive services, and high-capacity ferries, which provide cheap services (compare Yang *et alii* [22]).

In spite of the liberalization wave, we find it appropriate to assume that only two services are provided, hence (implicitly) that only two firms are active in the sector. This hypothesis reflects the circumstance that, very often, transport markets are imperfectly contestable and remain concentrated after being opened up to competition. Important barriers to entry are associated with schedule jockeying phenomena that are typical of markets where timing is a key determinant of consumption decisions.[3]

Furthermore, the assumption that two services are supplied makes our findings more apparent and intuitive. It helps catch the difference between classical vertical differentiation and the complex patterns of differentiation that we identify. Noticeably, the latter arise because, once the possibility of using both services is considered, three options are in fact available to users, despite only two services being supplied.

Not only, in a ferry market in which passengers can take services from all operators, heterogeneity in values of time deeply affects the way in which they allot across travel options (hence, the market segmentation). Also, it is found to be crucial in determining which interventions policy-makers should promote when pursuing redistribution purposes, depending upon the subset of passengers they wish to support. That is, in accordance with Small *et al.* [19],

[3] Schedule jockeying arises because, despite all services are scheduled, one firm may encounter the scheduled service of a competitor to arrive just a few moments before its own service. When the firm has no warranty from competitors' schedule, waiting passengers can be snapped by competitors applying sufficiently comparable fares. To discourage this behaviour, incumbents resort to root swamping, thereby creating a barrier to entry. Compare Klein and Moore [10] and Klein *et alii* [11] with regard to bus services. For a more general analysis, see Ehrhardt *et al.* [5].

"accounting for heterogeneity in values of time is important in evaluating constrained policies" (p.310).

Our study predicts that policy-makers should not insist on making the more expensive service more frequent because this does not represent the ideal policy intervention for any passenger. A cut in the price of that service is a limitedly powerful instrument, in turn. Policies concerning the price and, especially, the frequency of the cheaper service are more useful, overall.

To be more specific, when few connections are supplied and policy-makers envisage a raise in frequency that is large relative to the cut in price, it is likely that passengers who patronize the cheap service are tourists, those who use both services commuters. Then, by promoting a raise in the frequency of the cheaper service, policy-makers will favour both little patient tourists and little flexible commuters, for whom this is the most desirable change. By contrast, making the cheaper service even less expensive is the best policy in the eyes of very patient tourists, whereas cutting the price of the more expensive service is the most appropriate tool to please flexible commuters. When numerous connections are supplied and policy-makers envisage a raise in frequency that is small relative to the cut in price, presumably, commuters patronize the cheap service, whereas tourists use both services. In that case, policy-makers should only focus on the cheap service. A price cut is especially effective in that it favours the whole group of commuters and, additionally, the most patient fraction of tourists. A frequency raise should only be preferred when priority is given to highly flexible tourists.

The remainder of the paper is organized as follows. In section 2, we present the model. In section 3, we analyse the individual demand and the ensuing allotment of passengers. In section 4, we illustrate the peculiar patterns of product differentiation that arise in ferry industries and the resulting market segmentation. In section 5, we first describe the impact of changes in price and frequency on market segmentation; we then draw policy insights and discuss redistribution aspects. We conclude in section 6.

2 The model

We begin by describing the stylized model that we adopt to develop the analysis.

Supply of transportation services We consider a domestic ferry industry that provides transportation services to connect islands with continental territories. The industry is liberalized but concentrated. Two operators, O_l and O_h, are active. O_i provides service $i \in \{l, h\}$ of quality q_i with frequency f_i at the unit price p_i. Specifically, this is the price of a single ticket, allowing the purchaser to use any of the connection that O_i provides. Alternatively, p_i can be interpreted as the unit fee associated with a fixed-price pass that involves no additional per-travel charge. We assume that $p_l < p_h$, without loss of generality. We denote $\Delta p = p_h - p_l$. On the other hand, f_l can be either larger or smaller than f_h. At the industry level, $f = \sum_{i=l,h} f_i$ connections are supplied. The average price is $p = \sum_{i=l,h} p_i f_i / f$, the average quality $q = \sum_{i=l,h} q_i f_i / f$. As it is actually the case for a number of small islands in countries like Italy, Greece and Scotland, we assume that, besides the ferry services, no other transportation mode is available.

Demand for transportation services Absent any substitute mode, the whole demand for transportation services is captive to the ferry industry. Individuals who travel can use both service l and service h. Assuming that their ideal departure time is uniformly distributed along the time interval between any two departures, they wait a time equal to $1/2f$, on average, to get a connection.[4] Passengers are identical in their valuation of quality but differ in their valuation of waiting time. The value of quality is denoted $\mu \in M \equiv [0, +\infty)$. For each individual, it captures the per-unit benefit that she derives from the quality of the concerned service. The value of time is denoted $\tau \in T \equiv [0, +\infty)$. For each individual, it captures the benefit that she foregoes (or, equivalently, the opportunity cost that she incurs) by waiting for a connection rather than engaging in her best alternative activity. The surplus of a passenger whose time value is $\tau \in T$ is given by

$$S(x_l, x_h; \tau) = U(x) - \sum_{i=l,h} \left(p_i - \mu q_i + \frac{\tau}{2f} \right) x_i. \tag{1}$$

$U(x)$ is the gross utility obtained from the use of a total of $x = \sum_{i=l,h} x_i$ service units. As usual, the utility function is such that $U' > 0$ and $U'' < 0$ and satisfies the Inada conditions. x_i indicates how many times the passenger travels on a ferry operated by O_i. The term $p_i - \mu q_i + (\tau/2f)$ measures the so-called *generalized price*. This is the total cost that the passenger incurs to use one unit of service $i = l, h$. It is given by the unit monetary price (p_i) net of the benefit associated with the service quality (μq_i) and augmented by the disutility of departure delay $(\tau/2f)$.[5]

Passenger classification Passengers can be classified into two types, according to the way in which they approach transportation services. The first type includes passengers who patronize one given service. This is referred to as type P (for *p*atron).[6] The second type includes passengers who take the first available connection, hence use services randomly. This is referred to as type R (for *r*andom).

[4] This assumption is argued to be highly realistic (see, for instance, Yang *et alii* [22]) and, because of this, widely used in transportation models.

[5] The formulation in (1) extends the one that Billette de Villemeur [3] uses with regards to an industry where only one service is offered to the case in which two services are offered and the benefits induced by their qualities are accounted for.

[6] Considering that patrons do know the frequencies at the time they travel, the assumption that the ideal departure time is uniformly distributed along the time interval between any two departures may look questionable as far as passengers of type P are concerned. In fact, as Mohring *et alii* [16] point out, that assumption reflects the circumstance that the probability of matching a connection depends upon the characteristics of the services rather than upon the patrons' actions.

3 Individual demand and passenger allotment

We now turn to study how the demand for ferry services is formed and the characteristics that it displays in the setting previously described. We proceed as follows. We first investigate how each individual, displaying a certain quality value and time value and belonging to a certain type of passengers, chooses the number of journeys to make, given the prices, qualities and frequencies faced in the market. We then take a global view and explore how, depending upon the market prices, qualities and frequencies, passengers with a different time value allot between supplied services and classify as belonging to some given type.

3.1 Individual demand

Passengers of type P Take first a passenger whose time value is $\tau \in T$ and patronizes service $i \in \{l, h\}$. Her demand for the latter is determined by maximizing the surplus function in (1) with respect to the argument x_i. This yields the following choice rule:

$$U'(x_i^P) = p_i - \mu q_i + \frac{\tau}{2f_i}. \tag{2}$$

According to (2), the number of trips x_i^P to be made is such that the benefit derived from the marginal trip equals the generalized price incurred for it. Although two ferry services are available in the market and could both be used, p_i, q_i and f_i appear to be the only attributes that matter in the passenger quantity choice. This is because service i is perceived as a *substitute* for service j so that, once the individual has decided to use it, the price that O_j, $j \neq i$, charges and the quality of the service and the number of connections that it provides have no bite in the choice of how much of service i will be used. Because of this, condition (2) is analogous to the choice rule that a passenger follows in a market where only one service is offered (compare Billette de Villemeur [3]).

Passengers of type R Passengers of type R are ready to use both of the supplied services and assumed to take the first available one. Alternatively, they can be interpreted as "representative travellers" who patronize either service, depending upon the ferry schedule that best fits their ideal departure time. In any case, for a passenger of this type exhibiting time value $\tau \in T$, the demand for ferry services is pinned down by the choice rule

$$U'\left(x^R\right) = p - \mu q + \frac{\tau}{2f}. \tag{3}$$

To interpret this condition, one is first to understand the circumstance that it reflects. In a fraction f_l/f of the cases, the passenger is carried by O_l and pays the price p_l for each trip. In the remainder fraction of cases, namely f_h/f, she is carried by O_h and charged the per-trip price p_h. Thus, out of a total of $x^R \equiv \sum_{i=l,h} x_i^R$ trips, she expects to use a quantity $x_i^R = x^R f_i/f$ of each service $i = l, h$. That is, the use of service i represents a quota f_i/f of the total use of ferry services. Accordingly, the passenger expects to pay the price $p_i^R = p_i f_i / f$ to use service i. Overall, passengers base their decisions on an estimate of the monetary amount to be paid that is given by $p \equiv \sum_{i=l,h} p_i^R$. This is computed by attaching to the price of each of the two services the probability of actually paying that price, which is expressed by the relative frequency of the concerned service. Similarly, the average quality is given by $q \equiv \sum_{i=l,h} q_i^R \equiv \sum_{i=l,h} q_i f_i / f$. In the light of this, condition (3) is now easily interpreted. It tells that a passenger with time value τ chooses the number of trips x^R such that the benefit obtained from the last such trip equals the *expected* generalized price $p - \mu q + (\tau/2f)$.

Unlike for passengers of type P, the demand depends upon the *expected* price p, the expected quality q and the *total* number of connections f supplied in the market. This evidences an important circumstance regarding passengers of type R. They perceive service l and h as *complements*, rather than substitutes.[7] They approach them as a unique *composite* good characterized by the triple of attributes (p, q, f).

3.2 Allotment between services and types

In the industry that we represent, each individual who decides to travel faces three possible options. First, she can patronize service l (*option* l). Second, she can patronize service h (*option* h). Third, she can use both services (*option* lh).[8] Individuals compare the three alternatives and select their preferred option. Provided the price is uniform for either service, this boils down to comparing the unit generalized prices that the various options induce, given the personal time value, and picking the option that yields the lowest such price. In what follows, we describe this process and the choices to which it leads.

Before turning to the formal investigation, it is useful to remark that, rather than the absolute quality levels, what matters in the decision process is the quality difference.[9] As long as the latter does not affect the unanimous ranking of services, the outcome of the process coincides with the one that would arise if quality levels were equal or the quality dimension were absent at all. In the analysis developed hereafter, we make reference to this scenario and neglect the quality attribute.

3.2.1 Option comparisons

Option h *vs* option l An individual whose time value is $\tau \in T$ prefers option l to option h if and only if

$$p_l + \frac{\tau}{2f_l} < p_h + \frac{\tau}{2f_h}.$$

[7]From (3) one can see that, given the quality of the services, passengers of type R benefit from any price decrease and from any frequency increase. However, a reduction in p_h is more or less welcome than a reduction in p_l depending upon how f_h and f_l compare. On the other hand, an increase in f_h is not as welcome as an increase in f_l, provided the former induces a bigger raise in p.

[8]We only list the options that are available to individuals who do decide to make a trip. This explains why a non-travelling option is not considered.

[9]This is coherent with the view that the way in which quality is normalized should not matter.

We first take $f_h > f_l$ and let $\Delta f = f_h - f_l$. Together with $p_h > p_l$, this means that service h is more frequent but also more expensive than service l. Rearranging the previous condition yields

$$\tau < \widetilde{\tau} \equiv 2 f_l f_h \frac{\Delta p}{\Delta f}. \tag{4}$$

$\widetilde{\tau}$ is the *marginal* value of waiting time with regards to the choice between option l and option h *i.e.*, the time value of the passenger who is just indifferent between the two services. Condition (4) shows that, given prices and frequencies, individuals prefer some certain option depending upon their time value. In particular, individuals with time value below $\widetilde{\tau}$ prefer option l. Waiting is not a big concern for them, hence they are available to use the less frequent service in exchange for the benefit of paying a lower price. Individuals with time value above $\widetilde{\tau}$ prefer option h. Waiting is very costly to them, hence they opt for the service that is more frequent, even if this requires spending more money.

We next take $f_h < f_l$. In this case, service l is not only cheaper than service h, but also more frequent. Then, obviously, individuals prefer option l no matter their time value.

Option lh ***vs*** **option** $i \in \{l, h\}$ An individual whose time value is $\tau \in T$ prefers option $i \in \{l, h\}$ to option lh if and only if

$$p_i + \frac{\tau}{2f_i} < p + \frac{\tau}{2f}.$$

First take $i = h$. Then, there is no value of τ in the feasible range for which this inequality holds. It means that all individuals prefer option lh to option h. This is a very natural outcome in that using both services involves not only facing more frequent connections but also paying a lower price, on average.

Next take $i = l$. Then, the inequality above is equivalent to

$$\tau < \widehat{\tau} \equiv 2 f_l \Delta p. \tag{5}$$

$\widehat{\tau}$ identifies the marginal value of the waiting time with regards to the choice between option lh and option l *i.e.*, the time value of the individual who is exactly as well off with option lh as with option l. In line with the first comparison, condition (5) confirms that, given prices and frequencies, individuals prefer one or the other option depending upon their time value. In particular, those with time value below $\widehat{\tau}$ are less concerned with wasting time than with saving money and prefer option l. By contrast, individuals whose time value exceeds $\widehat{\tau}$ look for more frequent connections, even if this involves spending more money, on average. Thus, they choose option lh.

3.2.2 Endogenous passenger allotment

The passenger allotment resulting in the market can be understood by considering condition (4) together with (5).

The first aspect we notice is that the marginal time value $\widetilde{\tau}$ is strictly larger than $\widehat{\tau}$. That is, the individual who is indifferent between option h and option l exhibits a strictly bigger opportunity cost than the individual who is indifferent between option lh and option l. Then, based on the outcome of the option comparisons, we deduce that $\widehat{\tau}$ represents the only relevant cut-off time value in the final allotment, which we illustrate hereafter.

Individuals whose time value is smaller than the cut-off level $\widehat{\tau}$ use the cheaper service only. This reflects the circumstance that they are especially concerned with the price aspect, whereas wasting time has relatively little importance for them. Because they patronize one specific service, these individuals classify as passengers of type P. Individuals whose time value is larger than the cut-off level $\widehat{\tau}$ use both service l and service h. This reveals that they are willing to pay more for the benefit of more frequent departures. Because they randomize over services, these individuals classify as passengers of type R. How these passengers are specifically allocated between services it depends upon the relative frequencies. Lastly, individuals whose time value exactly equals the cut-off level $\widehat{\tau}$ are as well off patronizing the cheaper service as using both services.

The very possibility that travellers derive their utility from using a bundle of services is not a fictitious artefact of our model. A similar behaviour has been already evidenced in several industries. It is actually the object of the "mix-and-match" literature where consumers are assumed to assemble necessary components, possibly bought from different sellers, into a system that is close to their ideal (see Matutes *et al.* [13], for instance). The way in which passengers of type R act in our model is reminiscent of that. They conveniently bundle ferry connections to obtain a transportation service that is closer to their ideal, particularly in terms of travel schedule. This also means that, despite not being formalized, the dimension relating to the ideal departure time does appear in our model through the behaviour of impatient passengers.[10]

From our previous analysis, it should be clear that the allotment of passengers with different time value is determined *endogenously* once operators have chosen their price-and-frequency policies. However, it is important to remark that frequency policies do not have the same impact as pricing policies. The way in which individuals allocate across the three available options and that in which they classify as belonging to some given type of passengers are driven by the sole relationship between prices. To check this, first take the cheaper service to be also more frequent $(f_l > f_h)$. In this case, everybody prefers option l to option h. However, individuals whose $\tau \in (\widehat{\tau}, +\infty)$ prefer option lh to option l. Next take the cheaper service to be

[10] Incidentally, one could conjecture that, as in mix-and-match models firms may decide to make their components compatible with the competitors' so as to profitably fine-tune assemblage by consumers, in our framework operators might have an interest in harmonizing travel schedules in order to take better advantage of impatient passengers bundling the two available services.

less frequent $(f_l < f_h)$. In this case, passengers whose $\tau \in (\widetilde{\tau}, +\infty)$ prefer option h to option l. However, everybody prefers option lh to option h and, as before, passengers whose $\tau \in (\widehat{\tau}, +\infty)$ prefer option lh to option l. Provided $\widehat{\tau} < \widetilde{\tau}$, $\widetilde{\tau}$ is irrelevant, as already seen. Therefore, whether f_l is larger or smaller than f_h, the same passenger allotment entails. The relationship between frequencies comes to be important as to how O_l and O_h share demand eventually. Sometimes, for particular price offers, the resulting passenger allotment and classification degenerate. To see this, it is useful to consider the limit case in which operators charge the same price. Then, option lh yields exactly the same monetary expense, in expectation, as each of the other two options but warrants more frequent connections. It is thus obvious that even individuals with very low time value prefer using both services. All individuals classify as passengers of type R as a consequence of the two operators adopting the same pricing policy.

4 Product differentiation and market segmentation

Resting on the previous findings, we can appraise how the ferry market is segmented when two services are supplied and each passenger can use them both.

Looking at the whole industry, the frequencies f_l and f_h may be considered as quality attributes, along with q_l and q_h. In fact, a greater frequency means a lower average waiting time for the representative traveller. Other interpretations are, of course, possible.[11] Whatever the interpretation, the attribute f_l, just like q_l, appears to characterize both the "aggregate product" composed by all services provided by O_l and the "aggregate product" composed by all transportation services offered in the ferry market. As the "aggregate product" composed by all services supplied by O_h is never preferred to any of the other "aggregate products," the attributes f_h and q_h end up mattering only to explain the demand for the "aggregate product" composed by the whole bulk of ferry services provided in the market.

Individuals approach the cheaper service as a *basic* good that displays "quality" f_l and is sold at the unit price p_l. By contrast, the more expensive service is only perceived to be useful as a component of the service bundle lh. In turn, the bundle is viewed as an *improved* good *i.e.*, a good of updated "quality" $f > f_l$ that is sold at a higher price $p > p_l$. This evidences an interesting point. Although frequency can be seen as capturing the *variety* dimension of the two supplied services *i.e.*, the multiplicity of "sub-products" (the single trips) that each service is composed of, it can always be interpreted also as a parameter reflecting vertical differentiation. In fact, good l and good lh are ranked unanimously in the population. Yet, due to the peculiar market segmentation, vertical differentiation operates differently in this framework.

Under "classical" vertical differentiation, the cheaper service would be provided with small

[11] In particular, the market demand in our model can be viewed as the sum of individual unitary demands in a model where each traveller uses the service that is closest to her ideal departure time. Along this line, frequencies obviously represent parameters of a horizontal-differentiation model. This does not preclude such parameters from being interpreted as quality attributes of "aggregate products," the latter being given by the whole bulk of services provided by O_l, the whole bulk of services provided by O_h, and the whole bulk of services provided in the market.

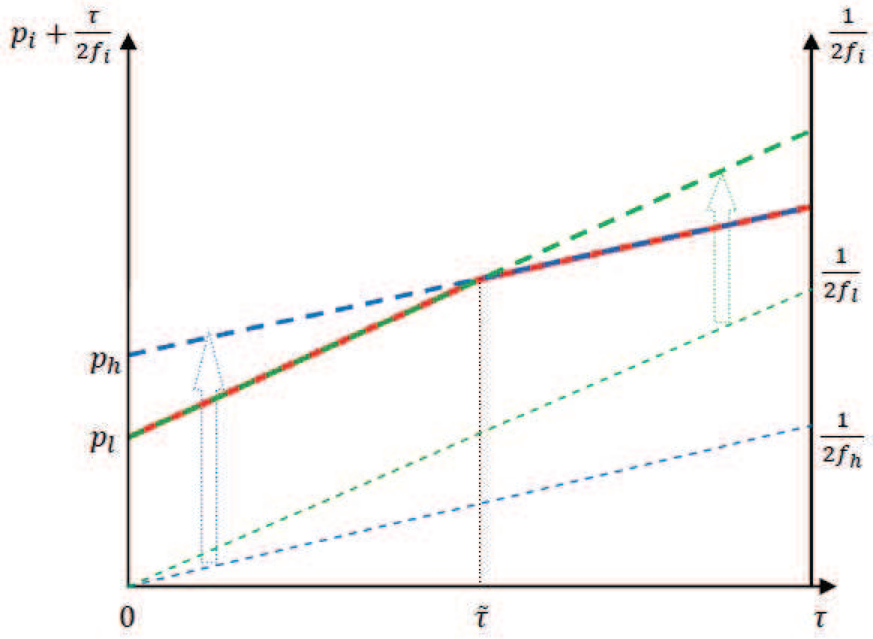

Figure 1: Market segmentation under classical vertical differentiation

frequency and patronized by low$-\tau$ passengers. The more expensive service would be provided with bigger frequency; it would represent a relevant option for passengers and be patronized by those with higher τ. The relevant cut-off time value would be $\widetilde{\tau}$, as defined in (4). Obviously, this would occur only if individuals could not use both services, in which case they would all behave as passengers of type P.[12]

The particular segmentation that arises in the ferry industry has also consequences on the impact that operators have on the market. Because the cheap service is considered to be both an autonomous good and a component of the bundle, O_l is active *vis-à-vis* the whole population of passengers. By contrast, in a vertically differentiated industry, it would only serve the segment $[0, \widetilde{\tau}]$ of the market. Furthermore, despite that the expensive service is not perceived as being an autonomous good, O_h faces a wider segment of passengers (namely, the range of time values $[\widehat{\tau}, +\infty)$) than it would in a vertically differentiated industry. Therefore, both operators have a more extensive impact on the market than they would under pure vertical differentiation.

The market segmentation that would emerge under vertical differentiation is represented graphically in Figure 1, where the green color is used for service l and the blue color for service h. The dashed thin lines represent the value of frequency to the patrons of the two services as an increasing function of their time value. The dashed thick lines represent the generalized prices associated with the two services. Each of them is obtained by shifting the one indicating

[12]In markets where two products are differentiated vertically in a standard manner, individuals purchase either one or the other product (not both) and it is optimal for firms to choose "extreme" policies. That is, one firm offers a high-quality product at a high price, the other offers a low-quality product at a low price (hence, $f_h > f_l$ together with $p_h > p_l$).

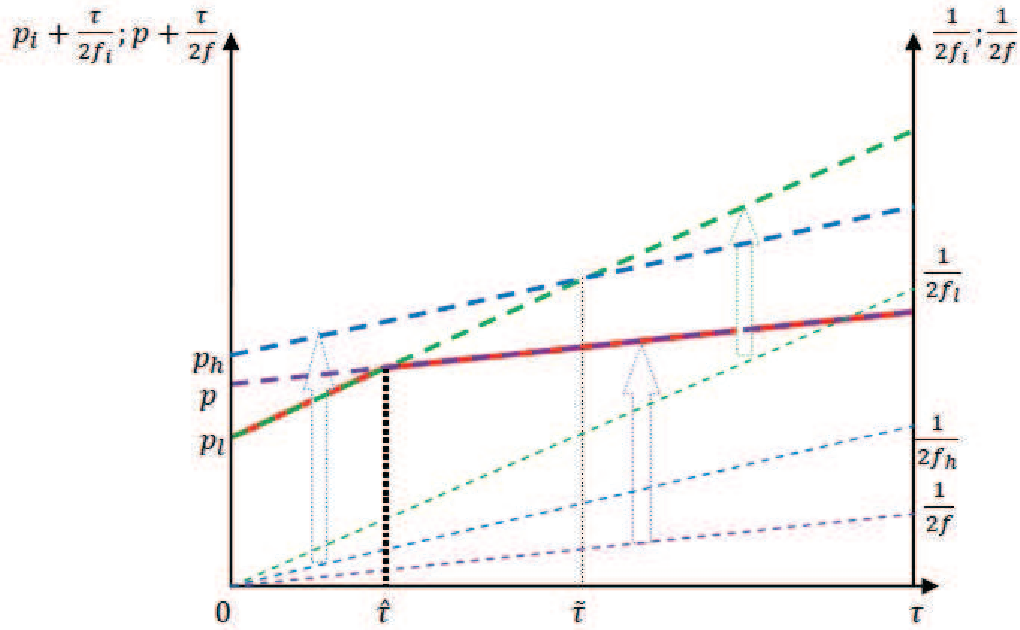

Figure 2: The ferry market segmentation with $f_h > f_l$

the value of frequency by an amount equal to the monetary price of the concerned service. The red broken line represents the generalized price for the whole population under the resulting market segmentation. Figure 2 provides a graphical illustration of the segmentation that arises, instead, in the ferry market. The graph is to be interpreted as that in Figure 1 except that, in addition, the purple color is used for the bundle.

To stress the contrast between vertical differentiation and ferry-service differentiation, the graph in Figure 2 is drawn for $f_h > f_l$. However, the particular market segmentation represented in Figure 2 still arises in the opposite case, in which the low-price service is also more frequent. Recall, indeed, that the passenger allotment between the basic and the improved good does not depend upon how frequencies compare. Passengers of type P patronize service l independently of how frequent its connections are. Yet, O_l is unable to attract the whole population of passengers, whether it offers more or less trips than O_h. This strikes one more difference with respect to a pure vertically differentiated setting. In the latter, if the cheaper service were also more frequent, then all passengers would prefer this service to the other. Consequently, absent capacity constraints, the industry would be a single-product monopoly, in fact. The case in which the cheap service is less frequent than the expensive service is graphically represented in Figure 3. By comparing with Figure 2, one immediately realizes that the market segmentation is the same in the two scenarios.

The fact that the passenger allotment does not depend upon the relationship between frequencies has a remarkable implication on the demand that operators face. No matter how rival prices and frequencies compare, demand is positive for *either* service. This involves that even an inefficient operator would be able to serve a portion of the market. A graphical representation is provided in Figure 4, where $g(\tau)$ denotes the density function associated with

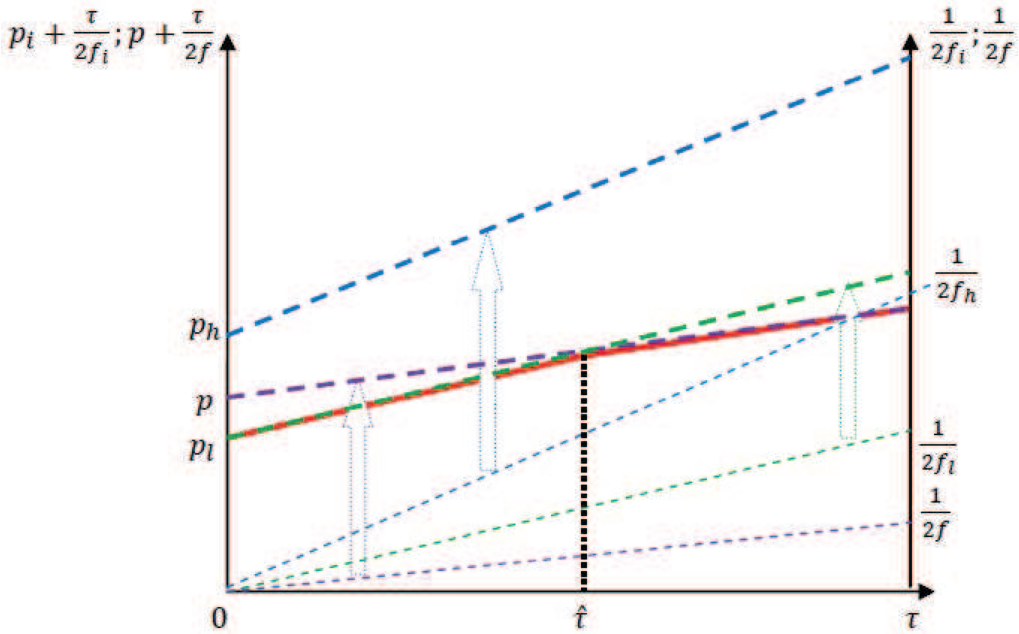

Figure 3: The ferry market segmentation with $f_l > f_h$

the distribution of time value in the population. The graphs to the left refer to the case in which the low-price service is less frequent, the graphs to the right to the case in which lower price comes along with higher frequency. On either side, the bottom graph represents the segmentation of the density between services, the green line regarding service l, the blue line service h. The market segment that would accrue to an inefficient operator is evidenced in the bottom graph to the right.

5 Price/Frequency changes and policy implications

We saw that the decisions made by O_l and O_h have a broader impact in the ferry market than they would in a vertically differentiated industry. We now investigate how variations in prices and in frequencies affect the market segmentation. This will enable us to draw insights about policy issues and discuss distributional implications.

We perform a very simple exercise of comparative statics. We let a certain price or frequency vary, all other variables being unchanged, and assess the impact that this has on the generalized price associated with the various travel options. We content ourselves with considering the direct effect of a variation in the concerned price/frequency. We abstract from the possibility of this change triggering indirect effects on the other variables, which could result from the strategic interactions between operators.[13]

We still neglect quality dimensions and the possibility of quality levels being varied. This is

[13]For instance, if operators compete *à la* Stackelberg, the leader internalizes the changes in the follower's price and/or frequency induced by variations in its own price and frequency. A detailed analysis can be found in Bergantino *et alii* [2].

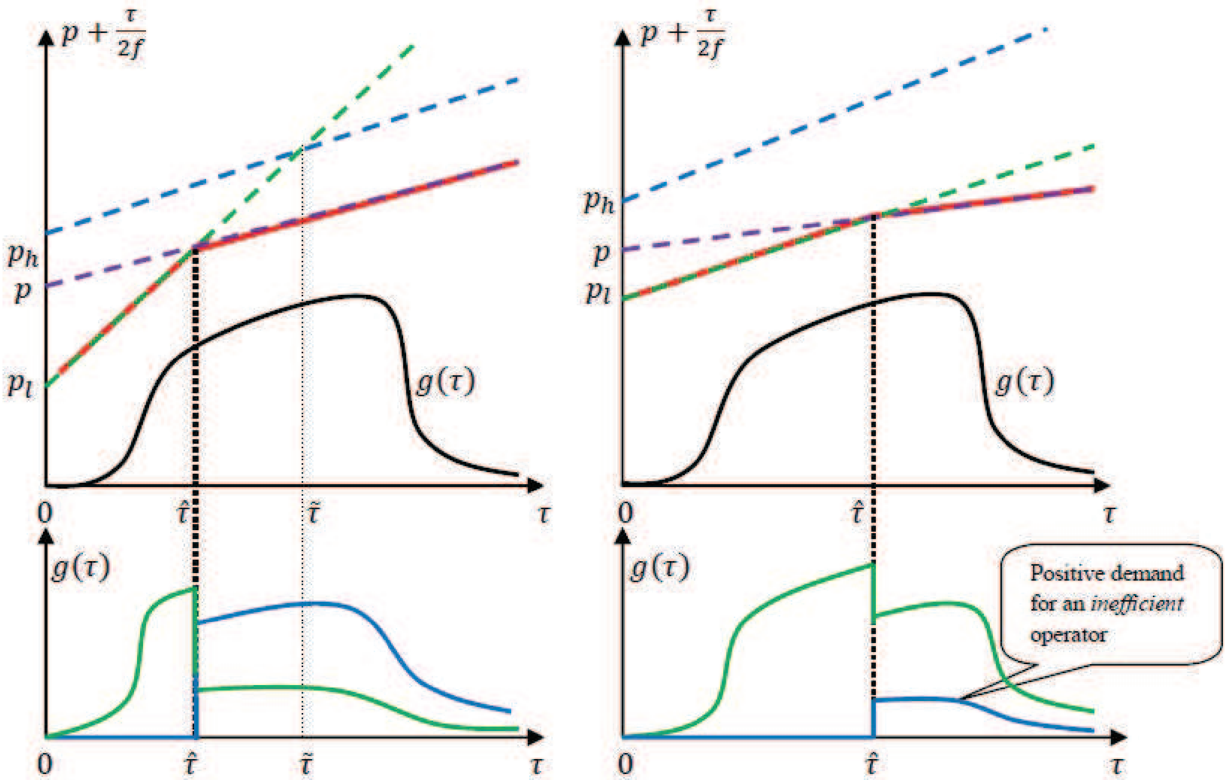

Figure 4: Segmentation of the ferry market and of the time-value density with $f_l < f_h$ (graphs to the left) and with $f_l > f_h$ (graphs to the right)

coherent with the practical circumstance that some quality aspects are not as flexible as price and frequency and can only be adjusted in a longer run. This is the case of service attributes, such as travel comfort and travel speed, that depend upon the very technical characteristics of the ferries.

5.1 Variations in p_l and f_l

We begin by exploring the impact of variations in the price and frequency proposed by O_l. Whether p_l or f_l is changed, the generalized price associated with option h is unaffected. Thus, we only need to analyse the consequences for option l and option lh.

First assume that the price of the cheap service is cut marginally. For all values of τ, the generalized price associated with option l and that associated with option lh decrease by 1 and f_l/f, respectively. Thus, the reduction is systematically more important for option l and independent of the time value for either option. Neither it depends upon the relationship between service frequencies. For the situation in which service l is less frequent than service h, an illustration is provided in the graphs to the left of Figure 5. The decrease in generalized price is captured, in the top graph, by the parallel downward shift in the green and in the purple line for option l and lh, respectively.

Overall, following to the reduction in p_l, the cut-off time value $\widehat{\tau}$ increases by $2f_l$. It means that some passengers shift from the bundle of services to option l, which they now find relatively

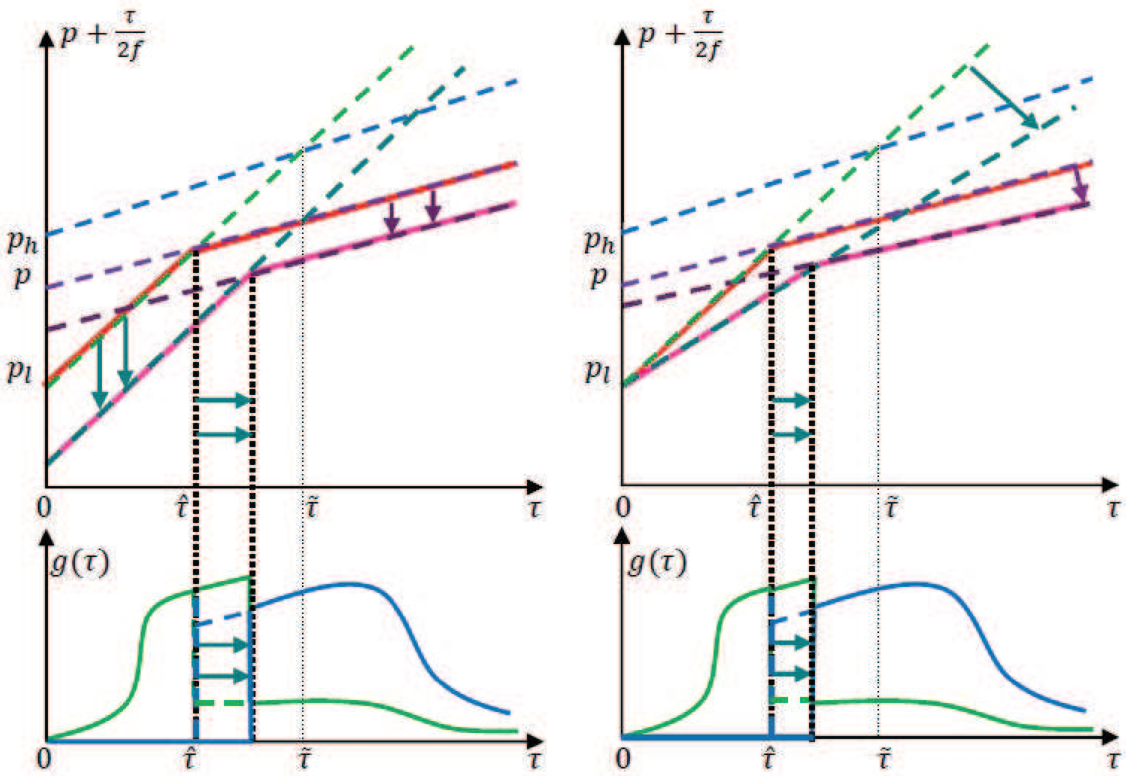

Figure 5: Effect of a decrease in p_l (graphs to the left) and of an increase in f_l (graphs to the right) for $f_h > f_l$

more convenient. The shift, which is represented in the bottom graph, is more important the larger f_l. The generalized price decreases for all passengers. In particular, it decreases by 1 for passengers of type P who still patronize service l, by f_l/f for passengers of type R who still use both services, by $(\tau - \widehat{\tau})\, f_h/2ff_l$ for passengers who were originally of type R and become of type P after the price cut. The change in generalized price for the whole population is represented by the shift from the red to the pink line in the top graph.

Next assume that the frequency of service l is raised marginally. The effects of this change are illustrated in the graphs to the right of Figure 5. For any given $\tau \in T$, the generalized price associated with option l decreases by an amount of $\tau/2f_l^2$. That is, it decreases for all $\tau > 0$, the reduction being more important the higher τ. Intuitively, further scheduling yields no benefit to individuals who do not mind to wait ($\tau = 0$) and an increasingly larger benefit to less patient individuals. This is represented by the downward rotation of the green line around the vertical intercept p_l in the top graph. Furthermore, for any given $\tau \in T$, the generalized price associated with option lh decreases by an amount of $(\tau + 2f_h\Delta p)\,/2f^2$. Hence, unlike for option l, the reduction occurs for all values of τ, including zero, as captured by the downward shift and rotation of the purple line in the top graph. This is explained by the circumstance that, as far as option lh is concerned, the raise in f_l triggers a reduction not only in the disutility of waiting time, which is equal to $\tau/2f^2$, but also in the expected monetary price, which is equal to $\Delta p f_h/f^2$. Specifically, the latter measures the expected saving that follows from the low price being paid more often. Because this saving is available to everybody, the generalized price is lower even for an infinitely patient individual. On the other hand, like for option l, the decrease in generalized price is more important the bigger τ. However, while the decrease is larger for option lh when τ is little, it becomes smaller for τ sufficiently big.[14]

In terms of market segmentation, the outcome of a marginal raise in f_l is similar to that of a cut in p_l. The cut-off time value increases, by $2\Delta p$ in this case, involving that a bigger portion of the population patronizes service l. The shift of passengers from the bundle to service l is represented in the bottom graph for $f_h > f_l$. The generalized price decreases for all passengers. In particular, it decreases by $\tau/2f_l^2$ for passengers of type P who still patronize service l, by $(\tau + 2f_h\Delta p)/2f^2$ for passengers of type R who still use both services, by $(\widehat{\tau} - \tau)f_h/2f_l f$ for passengers who were originally of type R and become of type P after the frequency raise. The change in generalized price for the whole population is represented by the jump from the red to the pink line in the top graph.

5.1.1 Policy implications

We learnt that all individuals gain from both a cut in p_l and a raise in f_l in that the generalized price they face reduces. The next step is to investigate when a cut in p_l is more beneficial than a raise in f_l and for which passengers this specifically occurs. At this aim, we contrast a price reduction equal to the marginal cut $\delta p_l = dp_l$ with a frequency increase $\delta f_l = \theta\,(df_l)$ *i.e.*, equal to the marginal increase df_l scaled by a positive parameter θ. This formalization allows us to account for the possibility of policy-makers envisaging changes of a different measure in the two variables. Both at this stage and in the sequel of the study, we only look at passengers who keep the initial type after the concerned policy variation. Instead, we neglect passengers whose type changes. This imposes no significant restriction on the analysis because, as long as variations in price/frequency are sufficiently little, the switch between types only occurs in a neighborhood of $\widehat{\tau}$.

Recall that a marginal decrease in p_l and a marginal increase in f_l reduce the generalized price associated with option l by 1 and $\tau/2f_l^2$, respectively. Thus, for a passenger of type P, whose time value is τ, a price cut δp_l is preferred to a frequency raise δf_l if and only if

$$\tau < 2\frac{f_l^2}{\theta}.$$

Further recall that a marginal decrease in p_l and a marginal increase in f_l reduce the generalized price associated with option lh by f_l/f and $(\tau + 2f_h\Delta p)/2f^2$, respectively. Thus, whether the price change δp_l has a smaller or bigger impact than the frequency change δf_l depends upon the time value, the two frequencies and the price wedge. Specifically, for a passenger of type R, the price reduction is more valuable than the frequency increase as long

[14] One can check that $\tau/2f_l^2$ is greater than $(\tau + 2f_h\Delta p)/2f^2$ if and only if τ exceeds the threshold $2f_l^2\Delta p/(2f_l + f_h)$, which is below $\widehat{\tau}$.

as

$$\tau < 2\left[\frac{f_l^2}{\theta} + f_h\left(\frac{f_l}{\theta} - \Delta p\right)\right].$$

Although, whatever the type of travellers, the gain from an increase in service frequency is larger the higher the time value, we distinguish two cases. The first arises when $f_l < \theta\Delta p$. Then $\widehat{\tau} > 2f_l^2/\theta$, meaning that passengers of type P do not have a homogeneous view of the benefits of the two variations. Those with especially low time value prefer a price cut, the others are better off with a frequency raise. Furthermore, $\widehat{\tau} > 2\left\{(f_l^2/\theta) + f_h\left[(f_l/\theta) - \Delta p\right]\right\}$. Hence, there is no passenger of type R for whom a price cut is more appealing. Frequency is so low that all passengers of that type have more to gain from an increase in f_l. The second case arises when, conversely, $f_l > \theta\Delta p$. In this case, $\widehat{\tau} < 2f_l^2/\theta$. Hence, patrons of service l all agree that a price reduction is preferable. Furthermore, $\widehat{\tau} < 2\left\{(f_l^2/\theta) + f_h\left[(f_l/\theta) - \Delta p\right]\right\}$ so that passengers of type R have different opinions about the desirability of the two variations. Service frequency is high enough that those who are not too impatient (*i.e.*, with time value sufficiently close to $\widehat{\tau}$) find a price cut more convenient. The others still prefer a frequency raise.

From the analysis here above, it emerges that the distributional implications of the policies depend upon the magnitude of θ, which measures the ratio between frequency and price variation $(\theta = \delta f_l/\delta p_l)$. In particular, they depend upon how θ compares with the threshold

$$\theta_l \equiv \frac{f_l}{\Delta p}.$$

This further evidences that the relative desirability of the policies δp_l and δf_l cannot be assessed unless considering the initial frequency of the cheap service and its price advantage over the rival service. If $\theta = \theta_l$, then passengers of type P all prefer the price cut, whereas passengers of type R all prefer the frequency raise. If $\theta > \theta_l$, then some passengers of type P, those with high τ, would also prefer the frequency raise to the price cut. By contrast, if $\theta < \theta_l$, then some passengers of type R, those with low τ, would also prefer the price cut to the frequency raise. In the particular case in which prices are nearly identical $(\Delta p \simeq 0)$, passengers of type P will always prefer a price cut (because $\theta_l >> 1$). On the other hand, as long as the frequency of the cheap service is very low $(f_l << \Delta p)$, passengers of type R will always prefer a raise in service frequency (because $\theta_l << 1$).

Moreover, the part of the population that a certain policy intervention can specifically target is likely to change, depending upon the concerned context. When few connections of service l are provided, a reasonable conjecture is that commuters classify as passengers of type R. They have low flexibility at adjusting their departure time, given the service schedule. Therefore, the bundle of the two services is likely more appropriate to match their needs. On the other hand, because tourists can more easily adjust their timetable to the schedule of some given service, they will rather classify as passengers of type P and opt for service l. Under this scenario, the introduction of a policy that promotes an increase in the frequency of the cheaper service will especially favour the inhabitants of the islands, who typically commute to the continental territories for work. In addition, it will particularly please less flexible tourists, who also welcome further scheduling of the cheap service. By contrast, the introduction of a policy that induces a cut in the price of the latter will particularly act in support of more flexible tourists, who are yet less numerous the lower the service frequency.

When many connections of service l are provided, it is perhaps more reasonable to conjecture that commuters be passengers of type P, instead. Because the service schedule is rich, they can afford to patronize the cheap service, despite being little flexible. On the other hand, one expects tourists to be more prone to behave as passengers of type R. Provided the number of connections is large and using the bundle allows passengers to take better advantage of them, adjustments to the specific schedule of service l are less attractive at least for the most flexible tourists. Under this scenario, the introduction of a policy that promotes an increase in the frequency of the cheap service will especially favour tourists and, in addition, little flexible commuters, who are yet less numerous the higher the service frequency. By contrast, the introduction of a policy that induces a cut in the price of the cheap service will act particularly in support of the commuters who can more easily adapt to the schedule of service l.

5.2 Variations in p_h and f_h

We now move to investigate the impact of variations in the price and frequency proposed by O_h. Whether p_h or f_h is changed, the generalized price associated with option l is unaffected and we can concentrate on the consequences for option h and option lh.

First take p_h to be decreased marginally. The impact of this variation is independent of how service frequencies compare. For $f_h > f_l$ it is illustrated in the graphs to the left of Figure 6. The price cut reduces the generalized price associated with option h by 1, that associated with option lh by f_h/f. Hence, the reduction is more important for option h. Moreover, it is constant across time values for either option. This can be visualized in the top graph, where the reduction in generalized price is represented by the parallel downward shift in the blue and purple line for option h and option lh, respectively.

As long as the price of service h remains larger than that of service l, the reduction in p_h does not change the result that some passengers choose option l, others option lh, whereas no passenger chooses option h. Following to the reduction in p_h, the cut-off time value $\widehat{\tau}$ decreases by $2f_l$, hence the decrease is more important the larger the frequency of the cheaper service. Some passengers shift from service l to the bundle of services, which they now find relatively more palatable. This shift is visible in the bottom graph. Noticeably, the generalized price does not decrease for all passengers. Only the passengers who were of type R before the change, together with those who become such after the change, gain from a cut in p_h. Specifically, the former obtain a gain of f_h/f, the latter a gain of $(\tau - \widehat{\tau})\, f_h/2f_l f$. The variation in generalized price for the whole population of concerned passengers is represented by the shift from the red to the pink line in the top graph.

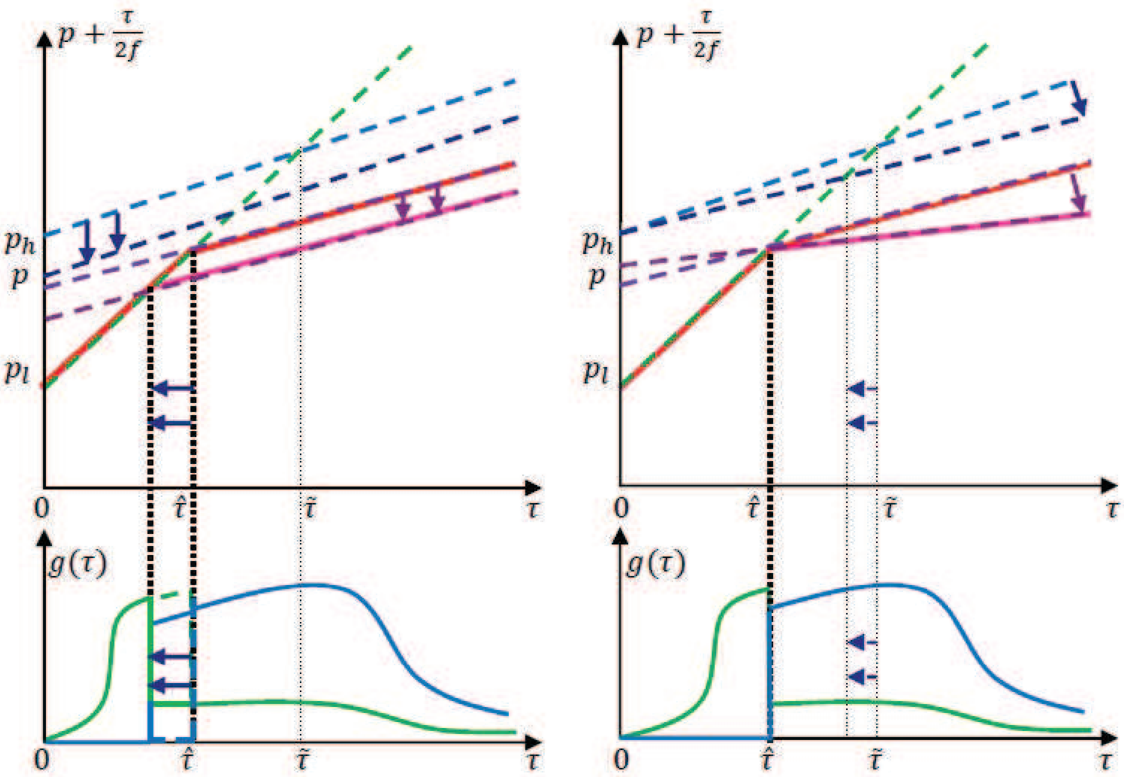

Figure 6: Effect of a decrease in p_h (graphs to the left) and of an increase in f_h (graphs to the right) with $f_h > f_l$

Next consider a marginal raise in the frequency of the expensive service. The effects of this change are represented in the graphs to the right of Figure 6, where again we take $f_h > f_{l.}$ For any given $\tau \in T$, the generalized price associated with option h decreases by an amount of $\tau/2f_h^2$. That is, it decreases for all $\tau > 0$, the reduction being more important the higher τ. This is represented by the downward rotation of the blue line around the vertical intercept p_h in the top graph. Furthermore, for any given $\tau \in T$, the generalized price associated with option lh changes by an amount of $(\tau - \widehat{\tau})/2f^2$. This reflects the circumstance that, while the disutility of waiting time decreases by $\tau/2f^2$, the expected price increases by $\widehat{\tau}/2f^2$ as the more expensive service is now used more often. Because of this, the generalized price associated with option lh either raises or declines, depending upon the time value. Specifically, it raises when $\tau < \widehat{\tau}$ and declines in the converse case, as evidenced by the rotation of the purple line in the top graph.

Overall, just as a cut in p_h, the raise in f_h does not change the result that some passengers choose option l, others option lh, whereas no passenger chooses option h. Besides, the raise in f_h does not affect the cut-off time value $\widehat{\tau}$. It means that making the expensive service more frequent does not have any impact in terms of market segmentation, as the bottom graph illustrates. Moreover, it only benefits passengers of type R, whose gain is represented by the jump from the red to the pink line in the top graph.

5.2.1 Policy implications

To investigate the implications of a change in p_h or f_h, for the reasons previously explained, we still neglect the group of passengers who switch from one type to the other after the concerned change is made. Here, such passengers are those with opportunity cost close to the cut-off value $\widehat{\tau}$, who move from service l to the service bundle after a cut in p_h. We thus focus on passengers who use both services already in the first place. Provided these are the only individuals who benefit from both a cut in p_h and a raise in f_h, we investigate their ordering of preferences over these two policies.

In the same vein as above, we contrast a price reduction equal to the marginal cut $\delta p_h = dp_h$ with a frequency increase $\delta f_h = \theta (df_h)$ *i.e.*, equal to the marginal increase df_h scaled by the positive parameter θ. Hence, we now have $\theta = \delta f_h / \delta p_h$. We saw that, for passengers of type R, a marginal cut in p_h and a marginal increase in f_h reduce the generalized price by f_h/f and $(\tau - \widehat{\tau}) / 2f^2$, respectively. Thus, the price cut δp_h is preferred to the frequency raise δf_h if and only if

$$\tau < \widehat{\tau} + 2\frac{f_h f}{\theta}.$$

Because the benefit from a cut in p_h is constant whereas that from a raise in f_h increases with the time value, it is very intuitive that a price cut is more welcome to sufficiently patient individuals (those with $\tau \in (\widehat{\tau}, \widehat{\tau} + 2f_h f/\theta))$, whereas very impatient individuals (those with $\tau > \widehat{\tau} + 2f_h f/\theta)$ are better off with a frequency raise. Importantly, this group of passengers is thinner the more the connections already supplied not only by O_h but also at the industry level.

To draw conclusions in terms of distributional effects, it is useful to come back to the conjectures previously made about the identity of passengers of different types. When services are little frequent so that tourists are likely passengers of type P and commuters passengers of type R, a policy that promotes a cut in the price of the more expensive service favours little patient tourists together with reasonably flexible commuters. On the other hand, a policy that leads to a raise in the frequency of service h, while benefiting all commuters, is an effective tool to support, in particular, those who are little flexible, who represent a large portion of the whole population of commuters if frequency is low. When services are very frequent so that commuters likely classify as passengers of type P and tourists as passengers of type R, policy-makers should cut p_h if they wish to favour not too inflexible commuters and/or sufficiently patient tourists. By contrast, if they aim at encouraging very impatient tourists to visit the islands, then they should rather promote further scheduling for service h. However, in that case, they should be aware that, as the number of connections is already large in the industry, the targeted population is quite thin.

In general, as compared to policies interventions regarding the price/frequency of the cheap service, those concerning the price/frequency of the expensive service come out to be less effective. Noticeably, they cannot be used to benefit the whole population of passengers, unless all individuals classify as passengers of type R.

5.3 Overall policy ranking

So far, we have compared and ordered policies two by two, initially focusing on those that concern price and frequency of the cheap service and subsequently looking at those that concern price and frequency of the expensive service. This approach is especially useful as long as policy-makers are restricted to target either the former or the latter. Nonetheless, when the four policy options previously illustrated are all at hand, it only provides a partial view. Actually, in that case, policy-makers might rather wish to understand which interventions would favour most the different groups of passengers. This would enable them to pick the exact policy that would best support the specific group of passengers they wish to protect. At this aim, it is useful to construct the full ranking of policies from the passenger perspective. We hereafter describe this ranking, beginning with passengers of type P and then turning to passengers of type R.

5.3.1 Passengers of type P

We saw that passengers of type P are affected neither by changes in p_h nor by changes in f_h. Thus, as far as these passengers are concerned, only policies concerning p_l and f_l are relevant. We identified two possible cases, depending upon the magnitude of θ, the ratio between the envisioned price and frequency change. For $\theta < \theta_l$, the most patient passengers (those with $\tau \in [0, 2f_l^2/\theta))$ prefer the price reduction; the others rather welcome more frequency. For $\theta > \theta_l$, all travellers of type P prefer the frequency raise, instead. Resting on this outcome, a clear insight can be drawn. If policy-makers wish to favour passengers of type P, then rendering the cheap service even less expensive is always an effective strategy as there is always a fraction of passengers for whom this is the best option. A raise in its frequency becomes more desirable if the number of connections is little ($\theta < \theta_l$) and most of the travellers of type P are conjectured to have a time value close to the cut-off level $\widehat{\tau}$.

5.3.2 Passengers of type R

As compared to a raise in f_h, a raise in f_l triggers a definitely bigger benefit for passengers of type R through more important a reduction in expected price. On the other hand, a cut in p_l is more convenient for them than a cut in p_h if and only if

$$\theta_l > \theta_h \equiv \frac{f_h}{\Delta p},$$

In this case, as the cheaper service is also more frequent than the other, passengers save money more often following to a reduction in p_l. Moreover, policy-makers should prefer the cut in p_l to the raise in f_h whenever they wish to support sufficiently flexible passengers *i.e.*, those with $\tau < \widehat{\tau} + 2f_l f/\theta$. Otherwise, a raise in f_h appears to be more appropriate a tool. Besides, whether a cut in p_l should be preferred to a raise in f_l it depends upon the magnitude of f_l, hence of θ_l. As long as service l is little frequent $(\theta > \theta_l)$, a raise in f_l is in favour of all passengers of type

R. When service l is not that rare $(\theta < \theta_l)$, a cut in p_l supports sufficiently flexible passengers *i.e.*, those with $\tau < 2\{(f_l^2/\theta) + f_h[(f_l/\theta) - \Delta p]\}$. By contrast, highly inflexible passengers would still welcome a raise in f_l. Whether passengers of type R prefer a raise in f_l to a cut in p_h, it depends upon how frequent connections are in the industry and, in particular, how frequent service h is. Not surprisingly, when total frequency is so little that

$$\theta > \widehat{\theta} \equiv \frac{f}{\Delta p},$$

everybody is better off with a raise in f_l. When, conversely, total frequency is big $(\theta < \widehat{\theta})$, the policy ranking is driven by the amount of connections that O_h provides. As long as f_h is low $(\theta > \theta_h)$, everybody is still better off with a raise in f_l as the monetary saving associated with a cut in p_h would be obtained little often. Otherwise $(\theta < \theta_h)$, a raise in f_l is more desirable only for little flexible passengers, those with $\tau > 2f_h[(f/\theta) - \Delta p]$, whereas the others do prefer a cut in p_h. Lastly, a cut in p_h should be preferred to a raise in f_h if passengers with $\tau < 2\{(f_h^2/\theta) + f_l[\Delta p + (f_h/\theta)]\}$ are to be favoured. By contrast, a raise in f_h is to be chosen in support of passengers with larger τ.

Putting everything together, it is possible to construct the complete ranking of policy changes for passengers of type R. This is done in Appendix A, where it is shown that eight cases are relevant, depending upon how θ compares with the thresholds θ_l, θ_h and $\widehat{\theta}$. Here we content ourselves with synthesizing the general lesson that the analysis delivers. As long as policy-makers wish to favour passengers of type R, in most of the cases, the best option is to raise the frequency of the cheaper service (δf_l). There are two exceptions though, both concerning sufficiently flexible individuals. First, when the cheaper service is also more frequent and the envisaged raise in frequency is little enough relative to the envisaged price cut ($\max\{\theta, \theta_h\} < \theta_l$), a reduction in price (δp_l) is the best strategy to favour individuals with $\tau \in (\widehat{\tau}, 2\{(f_l^2/\theta) + f_h[(f_l/\theta) - \Delta p]\})$. Second, when the expensive service is more frequent and the envisaged raise in frequency is neither too little nor too large relative to the envisaged price cut ($\theta_l < \theta < \theta_h$), a reduction in the price of the more expensive service (δp_h) is the best strategy to favour individuals with $\tau \in (\widehat{\tau}, 2f_h[(f/\theta) - \Delta p])$.

6 Concluding remarks

We proposed a very simple model to study passenger behaviour and derive policy implications in multi-service transportation markets, specifically referring to ferry markets as a good illustration of the contexts of our interest. We captured two main characteristics. First, passengers are heterogeneous in terms of opportunity cost of waiting time, whereas they display unanimous appreciation of quality aspects. Secondly, they do not necessarily need to patronize one specific service and may rather use *all* provided services. Core thrust of the work was to show that, under these circumstances, complex patterns of product differentiation arise, which are reminiscent of, but do not fully reflect, classical vertical differentiation.

Under vertical differentiation, consumers with low valuation for quality purchase the low-price low-quality product, those with high valuation for quality purchase the high-price high-quality product. In the markets we have considered, passengers with low time value always patronize the cheaper service, whether it is more or less frequent. Those with high time value use a bundle that comprehends *both* available services so that, in the end, the operator that charges the lower price carries passengers with any possible time value. With respect to the cheap service, the bundle represents a more expensive and more frequent service. Hence, it is the counterpart for the high-price high-quality product of a classical vertically differentiated setting, with the additional peculiarity that it accrues to a wider portion of the market. The more expensive service is solely purchased as a component of the bundle and, as such, it is consumed even when it is less frequent. Demand is thus positive even for an operator that provides a high-price low-frequency service.

Our investigation delivers a few policy implications. Of course, they should be taken with the necessary qualifications. For a more complete view and a clearer understanding of the economic interactions in the considered settings, a more structured analysis would be required, in which the supply side of the market would not be treated as a black box. Nevertheless, the hints that our analysis provides should be viewed as a preliminary step toward the elaboration of sound policy recommendations for liberalized transport sectors.

First of all, it is not apparent that price-and-quality regulatory mechanisms, such as that characterized by Bergantino *et alii* [2], could be adopted for partial regulation of ferry oligopolies, unless specific adjustments are performed. The reason for this is that, in oligopolistic transport markets, frequency can be loosely assimilated to a standard quality dimension only at the aggregate level. A natural research question would then be how price and (especially) frequency should be regulated in those settings.

Secondly, in ferry sectors that used to be served by public monopolists and have been recently opened up to competition, as it is the case in Italy, public operators are not in a position to crowd out private competitors. This result is to be contrasted with one that is recurrent in the domain of studies about mixed oligopolies. In the majority of the latter, profit-maximizing suppliers have room only if they are strictly more efficient than public operators. Estrin *et al.* [6] prove this outcome with regard to mixed markets in which quality is not a concern.[15] Interestingly, the peculiar market segmentation rules out the possibility of public monopolies persisting in the environments of our interest.

The flip side of the coin is that inefficient entry might occur in liberalized sectors with incumbents overstaying. This mismatches the wisdom, sometimes too easily received, that liberalization promotes efficiency by attracting cost-effective operators in economically wasteful monopolies. This aspect is particularly relevant in the setting that we focus on, provided all

[15] Technically speaking, the outcome we mention in the text follows from the relative position of the reaction curves of welfare-maximizing and profit-maximizing agents. See also Nett [17] for a presentation of the so-called *Cournot paradox* in mixed oligopolies where a homogeneous good is offered.

active firms (included a potentially inefficient entrant) have a wider impact on the market than they would in a classical vertically differentiated framework. Our model suggests that, as long as price competition remains soft because private entrants are inefficient, public operators have the possibility of maintaining direct control over the whole market. On the other hand, when public suppliers are not price aggressive, they can still boost "quality" (*i.e.*, frequency) *vis-à-vis* a fraction of the population that is larger than under standard vertical differentiation.

Importantly, our study delivers a few insights on the distributional impact that public interventions concerning the price and the frequency of the available services would have. Hence, it suggests which policies (whether price cuts or frequency raises) should be targeted when policy-makers wish to favour some given group of passengers.

A first neat lesson is that, unless for practical reasons other options are unavailable, policy-makers should not insist on additional scheduling of the expensive service because a raise in the frequency of the latter does not represent the ideal policy intervention for any passenger. Perhaps a bit surprisingly, also a cut in the price of the more expensive service is a limitedly powerful instrument. Policies concerning the price and, especially, the frequency of the cheaper service are definitely more useful, overall. To draw more specific conclusions about the appropriateness of the three latter policies, one cannot spare reference to two core elements *i.e.*, the amount of connections of the cheap service that are being supplied in the market when the public intervention takes place, and the relative magnitude of the price and frequency changes that policy-makers envisage.

When few connections are supplied and policy-makers envisage a raise in frequency that is large relative to the cut in price, it is likely that passengers who patronize the cheap service are tourists, those who use both services commuters. Then, by opting for a raise in the frequency of the cheap service, policy-makers will favour both little patient tourists and little flexible commuters, for whom this is the most desirable change. By contrast, pricing interventions would rather work in support of passengers with low time value for each type. Specifically, making the cheap service even less expensive is the best policy in the eyes of very patient tourists, whereas cutting the price of the expensive service is the most appropriate tool to please flexible commuters. When numerous connections are supplied and policy-makers envisage a raise in frequency that is small relative to the cut in price, presumably, commuters patronize the cheap service, whereas tourists use both services. In that case, there is no passenger for whom interventions on the price and the frequency of the expensive service represent the best option. Hence, policy-makers should focus on the cheap service. A cut in the price of the latter is especially powerful an instrument: it favours the whole group of commuters and, additionally, the most patient fraction of tourists. A raise in the frequency of the cheap service should be preferred only in the event that priority is given to highly flexible tourists.

All the findings recalled above were obtained under the hypothesis that passengers face uniform prices. Nevertheless, various other kinds of tariff are adopted, in practice. In many instances, operators offer to passengers the possibility of subscribing a pass for a fixed fee plus an additional per-travel charge (which is typically lower than the last-minute tariff).

Accounting for this pricing policy would involve considering explicitly a second dimension of market segmentation, relating to the passenger ideal frequency (and departure time), on top of that relating to the time value, which we embodied in the model. Intuitively, the introduction of more sophisticated pricing policies might be the root to further complexities in the differentiation pattern. This is left for future research.

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A Policy ranking for passengers of type R

For the sake of shortness, we introduce the following definitions:

$$\begin{aligned}
\overline{\tau} &\equiv \widehat{\tau} + 2\frac{f_l f}{\theta} \\
\underline{\tau} &\equiv 2\left[\frac{f_l^2}{\theta} + f_h\left(\frac{f_l}{\theta} - \Delta p\right)\right] \\
\tau' &\equiv 2\left[\frac{f_h^2}{\theta} + f_l\left(\Delta p + \frac{f_h}{\theta}\right)\right] \\
\tau'' &\equiv 2f_h\left(\frac{f}{\theta} - \Delta p\right).
\end{aligned}$$

To construct the full ranking of policies for passengers of type R, we order $\widehat{\tau}$, $\overline{\tau}$, $\underline{\tau}$, τ' and τ'' as follows:

$$\begin{array}{lll}
\underline{\tau} < \widehat{\tau} < \overline{\tau} < \tau'' < \tau' & \text{for} & f_l < \theta\Delta p < f_l + \theta\Delta p < f_h \\
\underline{\tau} < \widehat{\tau} < \tau'' < \overline{\tau} < \tau' & \text{for} & f_l < \theta\Delta p < f_h < f_l + \theta\Delta p \\
\underline{\tau} < \tau'' < \widehat{\tau} < \overline{\tau} < \tau' & \text{for} & f_l < f_h < \Delta p < f_l + \Delta p \\
\tau'' < \underline{\tau} < \widehat{\tau} < \tau' < \overline{\tau} & \text{for} & f_h < f_l < \Delta p < f_l + \Delta p \\
\tau'' < \widehat{\tau} < \underline{\tau} < \tau' < \overline{\tau} & \text{for} & f_h < \Delta p < f_l < f_h + \Delta p \\
\tau'' < \widehat{\tau} < \tau' < \underline{\tau} < \overline{\tau} & \text{for} & f_h < \Delta p < f_h + \Delta p < f_l \\
\widehat{\tau} < \tau'' < \underline{\tau} < \tau' < \overline{\tau} & \text{for} & \Delta p < f_h < f_l < f_h + \Delta p \\
\widehat{\tau} < \tau'' < \tau' < \underline{\tau} < \overline{\tau} & \text{for} & \Delta p < f_h < f_h + \Delta p < f_l
\end{array}$$

Accordingly, we distinguish eight cases, which we hereafter explore. In so doing, we denote $\succ$ the passengers' ordering of preferences over the options δp_l, δf_l, δp_h and δf_h.

Case 1: $f_l < \theta\Delta p < f_l + \theta\Delta p < f_h$ In this case:

$$\theta_l < \theta < \theta_l + \theta < \theta_h,$$

where $\theta_h \equiv f_h/\Delta p$. With $\theta > \theta_l$, it is $\underline{\tau} < \widehat{\tau}$ so that all passengers prefer δf_l to δp_l. Moreover, $\underline{\tau} < \widehat{\tau} < \overline{\tau} < \tau'' < \tau'$ and the following ranking arises:

$$\begin{aligned}
\delta p_h &\succ \delta f_l \succ \delta p_l \succ \delta f_h \text{ for } \tau \in (\widehat{\tau}, \overline{\tau}) \\
\delta p_h &\succ \delta f_l \succ \delta f_h \succ \delta p_l \text{ for } \tau \in (\overline{\tau}, \tau'') \\
\delta f_l &\succ \delta p_h \succ \delta f_h \succ \delta p_l \text{ for } \tau \in (\tau'', \tau') \\
\delta f_l &\succ \delta f_h \succ \delta p_h \succ \delta p_l \text{ for } \tau > \tau'.
\end{aligned}$$

Case 2: $f_l < \theta\Delta p < f_h < f_l + \theta\Delta p$ In thic case:

$$\theta_l < \theta < \theta_h < \theta_l + \theta.$$

Again, with $\theta > \theta_l$, it is $\underline{\tau} < \widehat{\tau}$ so that all passengers prefer δf_l to δp_l. With $\theta_h < \theta_l + \theta$, we now have $\tau'' < \overline{\tau}$. Overall, $\widehat{\tau} < \tau'' < \overline{\tau} < \tau'$ so that:

$$\begin{aligned}
\delta p_h &\succ \delta f_l \succ \delta p_l \succ \delta f_h \text{ for } \tau \in (\widehat{\tau}, \tau'') \\
\delta f_l &\succ \delta p_h \succ \delta p_l \succ \delta f_h \text{ for } \tau \in (\tau'', \overline{\tau}) \\
\delta f_l &\succ \delta p_h \succ \delta f_h \succ \delta p_l \text{ for } \tau \in (\overline{\tau}, \tau') \\
\delta f_l &\succ \delta f_h \succ \delta p_h \succ \delta p_l \text{ for } \tau > \tau'.
\end{aligned}$$

Case 3: $f_l < f_h < \theta\Delta p < f_l + \theta\Delta p$ In this case:

$$\theta_l < \theta_h < \theta < \theta_l + \theta.$$

Again, with $\theta > \theta_l$, it is $\underline{\tau} < \widehat{\tau}$ so that all passengers prefer δf_l to δp_l. With $\theta > \theta_h$, we further have $\tau'' < \widehat{\tau}$ so that all passengers also prefer δf_l to δp_h. Moreover, $\widehat{\tau} < \overline{\tau} < \tau'$. Hence:

$$\begin{aligned}
\delta f_l &\succ \delta p_h \succ \delta p_l \succ \delta f_h \text{ for } \tau \in (\widehat{\tau}, \overline{\tau}) \\
\delta f_l &\succ \delta p_h \succ \delta f_h \succ \delta p_l \text{ for } \tau \in (\overline{\tau}, \tau') \\
\delta f_l &\succ \delta f_h \succ \delta p_h \succ \delta p_l \text{ for } \tau > \tau'.
\end{aligned}$$

Case 4: $f_h < f_l < \theta\Delta p$ In this case:

$$\theta_h < \theta_l < \theta$$

Again, with $\theta > \theta_l$, it is $\underline{\tau} < \widehat{\tau}$ so that all passengers prefer δf_l to δp_l. With also $\theta > \theta_h$, it is overall $\tau'' < \underline{\tau} < \widehat{\tau}$ so that all passengers also prefer δf_l to δp_h. Furthermore, with $\theta_l > \theta_h$, everybody prefers δp_l to δp_h. Moreover, $\widehat{\tau} < \tau' < \overline{\tau}$. Hence:

$$\begin{aligned}
\delta f_l &\succ \delta p_l \succ \delta p_h \succ \delta f_h \text{ for } \tau \in (\widehat{\tau}, \tau') \\
\delta f_l &\succ \delta p_l \succ \delta f_h \succ \delta p_h \text{ for } \tau \in (\tau', \overline{\tau}) \\
\delta f_l &\succ \delta f_h \succ \delta p_l \succ \delta p_h \text{ for } \tau > \overline{\tau}.
\end{aligned}$$

Case 5: $f_h < \theta\Delta p < f_l < f_h + \theta\Delta p$ In this case:

$$\theta_h < \theta < \theta_l < \theta_h + \theta.$$

Again, with $\theta_l > \theta_h$, it is $\tau'' < \widehat{\tau}$ so that all passengers also prefer δf_l to δp_h; furthermore, everybody prefers δp_l to δp_h. Moreover, $\widehat{\tau} < \underline{\tau} < \tau' < \overline{\tau}$. Hence:

$$\begin{array}{rcll}
\delta p_l & \succ & \delta f_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\widehat{\tau}, \underline{\tau}) \\
\delta f_l & \succ & \delta p_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\underline{\tau}, \tau') \\
\delta f_l & \succ & \delta p_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\tau', \overline{\tau}) \\
\delta f_l & \succ & \delta f_h \succ \delta p_l \succ \delta p_h & \text{for } \ \tau > \overline{\tau}.
\end{array}$$

Case 6: $f_h < \theta\Delta p < f_h + \theta\Delta p < f_l$ In this case:

$$\theta_h < \theta < \theta_h + \theta < \theta_l.$$

Again, with $\theta > \theta_h$, it is $\tau'' < \widehat{\tau}$ so that all passengers also prefer δf_l to δp_h; furthermore, with $\theta_l > \theta_h$, everybody prefers δp_l to δp_h. Moreiver, $\widehat{\tau} < \tau' < \underline{\tau} < \overline{\tau}$. Hence:

$$\begin{array}{rcll}
\delta p_l & \succ & \delta f_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\widehat{\tau}, \tau') \\
\delta p_l & \succ & \delta f_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\tau', \underline{\tau}) \\
\delta f_l & \succ & \delta p_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\underline{\tau}, \overline{\tau}) \\
\delta f_l & \succ & \delta f_h \succ \delta p_l \succ \delta p_h & \text{for } \ \tau > \overline{\tau}.
\end{array}$$

Case 7: $\theta\Delta p < f_h < f_l < f_h + \theta\Delta p$ In this case:

$$\theta < \theta_h < \theta_l < \theta_h + \theta.$$

Again, with $\theta_l > \theta_h$, everybody prefers δp_l to δp_h. With $\theta_l < \theta_h + \theta$, we have $\underline{\tau} < \tau'$. Overall, $\widehat{\tau} < \tau'' < \underline{\tau} < \tau' < \overline{\tau}$. Hence:

$$\begin{array}{rcll}
\delta p_l & \succ & \delta p_h \succ \delta f_l \succ \delta f_h & \text{for } \ \tau \in (\widehat{\tau}, \tau'') \\
\delta p_l & \succ & \delta f_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\tau'', \underline{\tau}) \\
\delta f_l & \succ & \delta p_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\underline{\tau}, \tau') \\
\delta f_l & \succ & \delta p_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\tau', \overline{\tau}) \\
\delta f_l & \succ & \delta f_h \succ \delta p_l \succ \delta p_h & \text{for } \ \tau > \overline{\tau}.
\end{array}$$

Case 8: $\theta\Delta p < f_h < f_h + \theta\Delta p < f_l$ In this case:

$$\theta < \theta_h < \theta_h + \theta < \theta_l.$$

Again, with $\theta_l > \theta_h$, everybody prefers δp_l to δp_h. With $\theta < \theta_h$, we come back to $\tau'' > \widehat{\tau}$. We have $\widehat{\tau} < \tau'' < \tau' < \underline{\tau} < \overline{\tau}$. Hence:

$$\begin{array}{rcll}
\delta p_l & \succ & \delta p_h \succ \delta f_l \succ \delta f_h & \text{for } \ \tau \in (\widehat{\tau}, \tau'') \\
\delta p_l & \succ & \delta f_l \succ \delta p_h \succ \delta f_h & \text{for } \ \tau \in (\tau'', \tau') \\
\delta p_l & \succ & \delta f_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\tau', \underline{\tau}) \\
\delta f_l & \succ & \delta p_l \succ \delta f_h \succ \delta p_h & \text{for } \ \tau \in (\underline{\tau}, \overline{\tau}) \\
\delta f_l & \succ & \delta f_h \succ \delta p_l \succ \delta p_h & \text{for } \ \tau > \overline{\tau}.
\end{array}$$